

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} .$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} .$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

- 1 Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find the value of each of the constants m and n for which

$$\mathbf{A}^2 + m\mathbf{A} = n\mathbf{I},$$

where \mathbf{I} is the identity matrix.

[4]

- 2 Show that

$$\frac{1}{1 - \cos \theta} - \frac{1}{1 + \cos \theta} \equiv 2 \operatorname{cosec} \theta \cot \theta.$$

[4]

- 3 Given that $p = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, express in its simplest surd form,

(i) p , [3]

(ii) $p - \frac{1}{p}$. [2]

- 4 A badminton team of 4 men and 4 women is to be selected from 9 men and 6 women.

- (i) Find the total number of ways in which the team can be selected if there are no restrictions on the selection. [3]

Two of the men are twins.

- (ii) Find the number of ways in which the team can be selected if exactly one of the twins is in the team. [3]

- 5 In this question, \mathbf{i} is a unit vector due east, and \mathbf{j} is a unit vector due north.

A plane flies from P to Q where $\overrightarrow{PQ} = (960\mathbf{i} + 400\mathbf{j})$ km. A constant wind is blowing with velocity $(-60\mathbf{i} + 60\mathbf{j})$ km h⁻¹. Given that the plane takes 4 hours to travel from P to Q , find

- (i) the velocity, in still air, of the plane, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ km h⁻¹, [4]

- (ii) the bearing, to the nearest degree, on which the plane must be directed. [2]

- 6 A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{4x+1}}$, and $(6, 20)$ is a point on the curve.

- (i) Find the equation of the curve. [4]

A line with gradient $-\frac{1}{2}$ is a normal to the curve.

- (ii) Find the coordinates of the points at which this normal meets the coordinate axes. [4]

- 7 (i) Use the substitution $u = 2^x$ to solve the equation $2^{2x} = 2^{x+2} + 5$. [5]
- (ii) Solve the equation $2\log_9 3 + \log_5(7y - 3) = \log_2 8$. [4]
- 8 (a) The remainder when the expression $x^3 - 11x^2 + kx - 30$ is divided by $x - 1$ is 4 times the remainder when this expression is divided by $x - 2$. Find the value of the constant k . [4]
- (b) Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

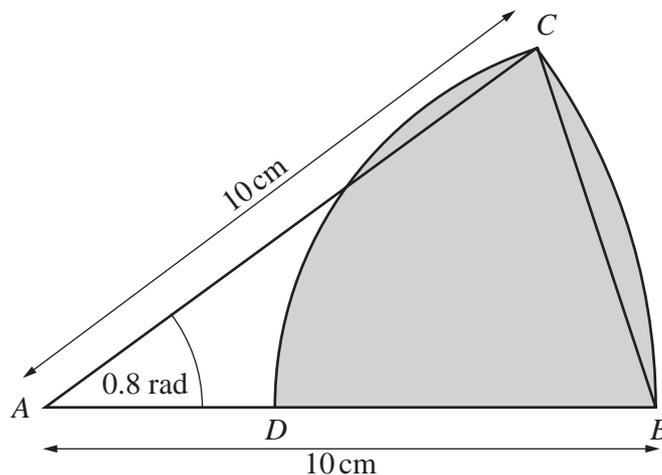
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x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4

The table shows experimental values of two variables, x and y .

- (i) Using graph paper, plot xy against x^2 . [2]
- (ii) Use the graph of xy against x^2 to express y in terms of x . [4]
- (iii) Find the value of y for which $y = \frac{83}{x}$. [3]

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The diagram shows a sector ABC of the circle, centre A and radius 10 cm , in which angle $BAC = 0.8$ radians. The arc CD of a circle has centre B and the point D lies on AB .

- (i) Show that the length of the straight line BC is 7.79 cm , correct to 2 decimal places. [2]
- (ii) Find the perimeter of the shaded region. [4]
- (iii) Find the area of the shaded region. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A curve has the equation $y = xe^{2x}$.

- (i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]
- (ii) Show that the y -coordinate of the stationary point of the curve is $-\frac{1}{2e}$. [3]
- (iii) Determine the nature of this stationary point. [2]

OR

- (i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{1 - 2\ln x}{x^3}$. [3]
- (ii) Show that the y -coordinate of the stationary point of the curve $y = \frac{\ln x}{x^2}$ is $\frac{1}{2e}$. [3]
- (iii) Use the result from part (i) to find $\int\left(\frac{\ln x}{x^3}\right)dx$. [4]

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